a sequence of public spending. Given initial aggregate endowment $W(t)$ *and individual endowment* $\{W(s,t)\}_{s=-\infty}^{t}$ *such that:*

$W(t) = B(t) + K(t)$ *et* $W(t) = \int_{-\infty}^{t} \lambda e^{(-\lambda(t-s))} W(s,t) ds,$

A competitive equilibrium (Ramse equilibrium) is composed of allocations $\{\{C(s,z), L(s,z)\}_{s=-\infty}^{+\infty}\}_{z=t}^{+\infty}$ *and* $\{K(z)\}_{z=t}^{+\infty}$*, of a fiscal polic arrangement* $\pi = \{T_\omega(s,z), T_k(s,z)\}_{s=-\infty}^{+\infty}, B(z)\}_{z=t}^{+\infty}$*, of a price s stem* $p = \{w(z), r(z)\}_{z=t}^{+\infty}$*, and*

- for an agent born on date $s < t$:

$$\int_t^{+\infty} e^{-(\delta+\lambda)(z-s)} \left(U_{C(s,z)} C(s,z) + U_{N(s,z)} N(s,z)\right) dz = e^{-(\delta+\lambda)(t-s)} U_{C(s,t)} W(s,t)$$

- for an agent to be born on date $s > t$:

$$\int_s^{+\infty} e^{-(\delta+\lambda)(z-s)} \left(U_{C(s,z)} C(s,z) + U_{N(s,z)} N(s,z)\right) dz = U_{C(s,s)} W(s,s)$$

and the feasibilit constraint :

$$\dot{K}(z) = Y(z) - dK(z) - C(z) - G(z)$$

for $z \in [t, +\infty[$

Proof of proposition 1:

We prove propositions 2 and 3, in order to prove proposition 1. First, we show that by construction, an allocation which satisfies the first-order conditions of the consumer's problem satisfies the implementability constraint (proposition 2).

Second, we show that an allocation that satisfies the implementability and the feasibility constraint can be decentralised as a competitive equilibrium in an age-dependent tax system (proposition 3). (We follow the same procedure as Chari and Kehoe [1998], and Gervais and Erosa [2002]).

Proposition : *B construction, an allocation $\left\{\{C(s,z), L(s,z), W(s,z)\}_{s=-\infty}^{+\infty}\right\}_{z=t}^{+\infty}$ which satisfies the first-order conditions of the consumer's problem satisfies the implementabilit constraint.*

Proof of proposition

We start writing the first-order conditions on date z for an individual born on date s. We assume that those are necessary and sufficient conditions and that the allocations are interior (assuming that the utility function is concave, the first-order conditions are sufficient and interiority is ensured by monotony and Inada conditions):

$$\mu(s,z) = U_{C(s,z)}$$
$$\mu(s,z)\hat{\omega}(s,z) = -U_{N(s,z)}$$
$$\dot{\mu}(s,z) = [\delta - \hat{r}(s,z)]\, \mu(s,z)$$

We build the consumer's intertemporal budget constraint:

$$\int_t^{+\infty} R_\lambda(z) \left[C(s,z) - \omega(z)\left(1 - T\omega(s,z)\right) N(s,z)\right] dz = W(s,t)$$

Verified for $z \in [t, +\infty[$.

Substituting the first-order conditions in the consumer's intertemporal budget constraint,we get the implementability constraint for an indiviual born on date $s < t$:

$$\int_t^{+\infty} e^{-(\delta+\lambda)(z-s)} \left[U_{C(s,z)} C(s,z) + U_{N(s,z)} N(s,z)\right] dz = e^{-(\delta+\lambda)(t-s)} U_{C(s,t)} W(s,t)$$

In the same way, we obtain the implementability constraint for an individual born on date $s > t$:

$$\int_s^{+\infty} e^{-(\delta+\lambda)(z-s)} \left[U_{C(s,z)} C(s,z) + U_{N(s,z)} N(s,z) \right] dz = U_{C(s,s)} W(s,s)$$

Proposition 3: *If an allocation* $\{\{C(s,z), L(s,z)\}_{s=-\infty}^{+\infty} K(z)\}_{z=t}^{+\infty}$ *satisfies the implementabilit and feasibilit constraints, then a price s stem p, a fiscal polic* π, *asset holdings* $\{W(s,z)\}_{s=-\infty}^{+\infty}\}_{z=t}^{+\infty}$, *and an allocation* $\{K(z)\}_{z=t}^{+\infty}$ *constitute, with the given allocation, a competitive equilibrium in a fiscal s stem where taxes are age-..mtute,sationi.e-.-p.si-.se-.-.-.-..-upi-.se--.xes.all.a-i-.cawit..d..........ho-.......-.-.s.......*

2) Given asset holdings $\{\{W(s,z)\}_{s=-\infty}^{+\infty}\}_{z=t}^{+\infty}$, we rewite the consumer's budget constraint taking into consideration the expressions of after-tax prices given by (1) and (2), then:

$$\dot{W}(s,z) = (\lambda + \hat{r}(s,z))\, W(s,z) + \hat{\omega}(s,z) N(s,z) - C(s,z)$$

Integrating and using the other first-order conditions of the consumer, we get:

$$\int_t^{+\infty} e^{-(\delta+\lambda)(z-s)} \left[U_{C(s,z)} C(s,z) + U_{N(s,z)} N(\right.$$

3 The solution to the government's problem

On date z, the government choses the optimal allocation of aggregate consumption and labor between individuals of all ages in order to maximize their instantaneous pseudo utility υ. Given this first step, the government chooses an optimal allocation path (for consumption, labor and capital) maximizing its objective function under a feasibility constraint.

We solve the government's static and dynamic problems each time using a standard hamiltonian.

- The static problem: On date z, the government allocates aggregate consumption and labor among individuals of all ages.

$V(C(z),N(z)) = \max_{\{C(z-n,z),N(z-n,z)\}_{n=0}^{+\infty}} \int_0^{+\infty} \left[e^{-\lambda n}\upsilon(C\left(z-n,z\right),1-N\left(z-n,z\right))\right] e^{(\rho-\delta)n} dn$

s.t.: $C\left(z\right) = \int_0^{+\infty} \lambda e^{-\lambda n} C\left(z-n,z\right) dn$ and $N\left(z\right) = \int_0^{+\infty} \lambda e^{-\lambda n} N\left(z-n,z\right) dn$

Given ψ_1 and ψ_2, the multipliers associated with the agregation constraints, the optimality conditions of the static problem are:

$$\upsilon_{C(z-n,z)} e^{(\rho-\delta)n} = -\psi_1\left(z\right)\lambda$$

$$\upsilon_{N(z-n,z)} e^{(\rho-\delta)n} = -\psi_2\left(z\right)\lambda$$

for $n \in [0;+\infty[$ and for $\delta \leq \rho < \delta + \lambda$.

The dynamic problem: Given the optimal allocations determined by the static problem, the government chooses an optimal allocation path for all z.

$$\max_{\{\{C(z),N(z)\},K(z)\}_{z=0}^{+\infty}} \int_0^{+\infty} \left\{V(C(z),1-N(z))\right\} e^{-\rho z} dz$$

s.t.: $\dot{K}\left(z\right) = F'_{K(z)}K(z) + F'_{N(z)}N\left(z\right) - dK(z) - C\left(z\right) - G\left(z\right)$

Given ψ, the multiplier associated with the feasibility constraint, the necessary conditions of the dynamic problem are:

$$V_{C(z)} = \psi(z)$$

$$V_{N(z)} = -\psi(z)\omega(z)$$

$$\dot{\psi}\left(z\right) = \left[\rho - F_K^{'}\left(z\right)\right]\psi\left(z\right)$$

The implementability constraint implies that the transversality condition is verified for each individual. Since the implementability constraint is part of the government's objective, the transversality condition is verified at the aggregate level.

Proposition 4: Both problems (static and dynamic) are re-united using the result by Benveniste and Sheinkman [1979]: $V(\cdot)$ is strictly concave and can

We use the expression:

$v_{C(z-n,z)} = [1+\chi(z-n)+\chi(z-n)\xi^{C}\,(z-n,z)]U_{C(z-n,z)} -$ ∂n

The labor income tax is derived from the ratio between private and social S between consumption and leisure:

$$\frac{-U_{N(z-n,z)}/U_{C(z-n,z)}}{-V_{N(z-n,z)}/V_{C(z-n,z)}} = \frac{\hat{\omega}(z-n,z)}{\omega(z-n,z)} = (1 - T_{\omega(z-n,z)})$$

We use the expression:
$v_{N(z-n,z)} = [1+\chi(z-n)+\chi(z-n)\xi^N(z-n,z)]U_{N(z-n,z)} - \frac{\partial}{\partial N(z-n,z)}\left[\chi(z-n)U_{C(-n,0)}W(-n,0)\right]$

On date $z = 0$:
$v_{N(-n,0)} = [1+\chi(-n)+\chi(-n)\xi^N(-n,0)]U_{N(-n,0)} - \frac{\partial}{\partial N(-n,0)}\left[\chi(-n)U_{C(-n,0)}W(-n,0)\right]$
Equivalently:
$v_{N(-n,0)} = [1+\chi(-n)+\chi(-n)\xi^N(-n,0)]U_{N(-n,0)} - \chi(-n)U_{C(-n,0)N(-n,0)}W(-n,0)$

On dates $z > 0$:
$v_{N(z-n,z)} = [1+\chi(z-n)+\chi(z-n)\xi^N(z-n,z)]U_{N(z-n,z)}$

We calculate the optimal labor income tax on date zero and derive the general expression for all dates $z \geq 0$. After simplifications, we obtain:

$$T_\omega(z-n,z) = \frac{\chi(z-n)\left(\xi^N(z-n,z) - \xi^C(z-n,z) \quad + Ini1\right.}{}$$