

Technical Appendix

Appendix C

After rearranging the integrals, aggregate human wealth is given by:

$$H^R(t) = \int_t^{\infty} \frac{1}{\lambda_P + \lambda_R} e^{-\frac{1}{\lambda_R}(t-s)} [a_R e^{-\alpha_R(t-s)}] \\ \left\{ \int_t^z e^{-\int_t^v [\hat{r}(v) + \frac{1}{\lambda_R}] dv} \hat{\omega}^R(z) [\exp^{-\alpha_R(z-t)}] E^R(z) N^R(z) dz \right.$$

I reorganize the integrals as:

$$H^R(t) = \int_t^{\infty} \frac{a_R}{\lambda_P + \lambda_R} e^{-(\frac{1}{\lambda_R} + \alpha_R)(t-s)} \left[\int_t^z e^{-\int_t^v [\hat{r}(v) + \frac{1}{\lambda_R} + \alpha_R] dv} \hat{\omega}^R(z) E^R(z) N^R(z) dz \right] ds$$

with:

$$\lim_{z \rightarrow \infty} e^{-\int_t^z [\hat{r}(v) + \frac{1}{\lambda_R} + \alpha_R] dv} \hat{\omega}^R(z) E^R(z) N^R(z) = 0$$

and:

$$\int_t^{\infty} \frac{a_R}{\lambda_P + \lambda_R} e^{-(\frac{1}{\lambda_R} + \alpha_R)(t-s)} ds = 1$$

$$\frac{a_R}{\lambda_P + \lambda_R} \frac{1}{\frac{1}{\lambda_R} + \alpha_R} = 1$$

The term in braces is independent of s . Differentiating with respect to time yields:

$$\dot{H}^R(t) = \left(\hat{r}(t) + \frac{1}{\lambda_R} \right) H^R(t)$$

Appendix E

In the steady state,

When $\dot{H}^R(t) = 0$,

$$(\hat{r} + \alpha_R - \delta) C^R = \left(\delta + \frac{1}{\lambda_R} \right) \left(\frac{1}{\lambda_R} + \alpha_R \right) W^R$$

As a result:

$$\delta - \alpha_R < \hat{r}$$

The concavity of F implies that:

$$F(K) > rK$$

which gives:

$$C^R + C^P + rT_k K > rK$$

or equivalently:

$$\left(\delta + \frac{1}{\lambda_R}\right)(K^R + H^R) > \hat{r}K - C^P$$
$$\left(\delta + \frac{1}{\lambda_R} - \hat{r}\right)K^R + \left(\delta + \frac{1}{\lambda_R}\right)H^R + (C^P - \hat{r}K^P) > 0$$

(22)

For this inequality to hold, two sufficient conditions are:

Appendix F

Conditions for the strict concavity of V

Recall that V is described as follows:

I obtain:

$$\begin{aligned}
\frac{{}^2\mathbf{v}}{C^R(z)} &= \frac{\mathbf{v}_{C^R(z, n, z)}}{C^R(z)} \\
&= \chi(z, n) \frac{\xi^{C^R(z, n, z)}}{C^R(z)} U_{C^R(z, n, z)} + \left(1 + \chi(z, n) + \chi(z, n) \xi^{C^R(z, n, z)}\right) \frac{U_{C^R(z, n, z)}}{C^R(z)} \\
&= \chi(z, n) \frac{\xi^{C^R(z, n, z)}}{C^R(z)} U_{C^R(z, n, z)} + \left(1 + \chi(z, n) + \chi(z, n) \xi^{C^R(z, n, z)}\right) \frac{{}^2U(\cdot)}{C^R(z)}
\end{aligned}$$

Therefore,

$$\frac{{}^2\mathbf{v}}{C^R(n)} < 0 \text{ if } \chi(z, n) \frac{\xi^{C^R(z, n, z)}}{C^R(z)} U_{C^R(z, n, z)} + \left(1 + \chi(z, n) + \chi(z, n) \xi^{C^R(z, n, z)}\right) {}^2U(\cdot)$$

In a similar way,

$$\frac{{}^2\mathbf{v}}{L^R(n)} < 0 \text{ if } \left\{ \chi(z, n) \frac{\xi^{L^R(z, n, z)}}{L^R(z)} U_{L^R(z, n, z)} + \left(1 + \chi(z, n) \chi(z, n) \xi^{L^R(z, n, z)}\right) \frac{{}^2U(\cdot)}{L^R(z)} \right\} < 0$$

If those two last conditions are satisfied, \mathbf{v} is concave.