

A **heap** is a widely used data structure.

Heapsort algorithm (Williams 1964)

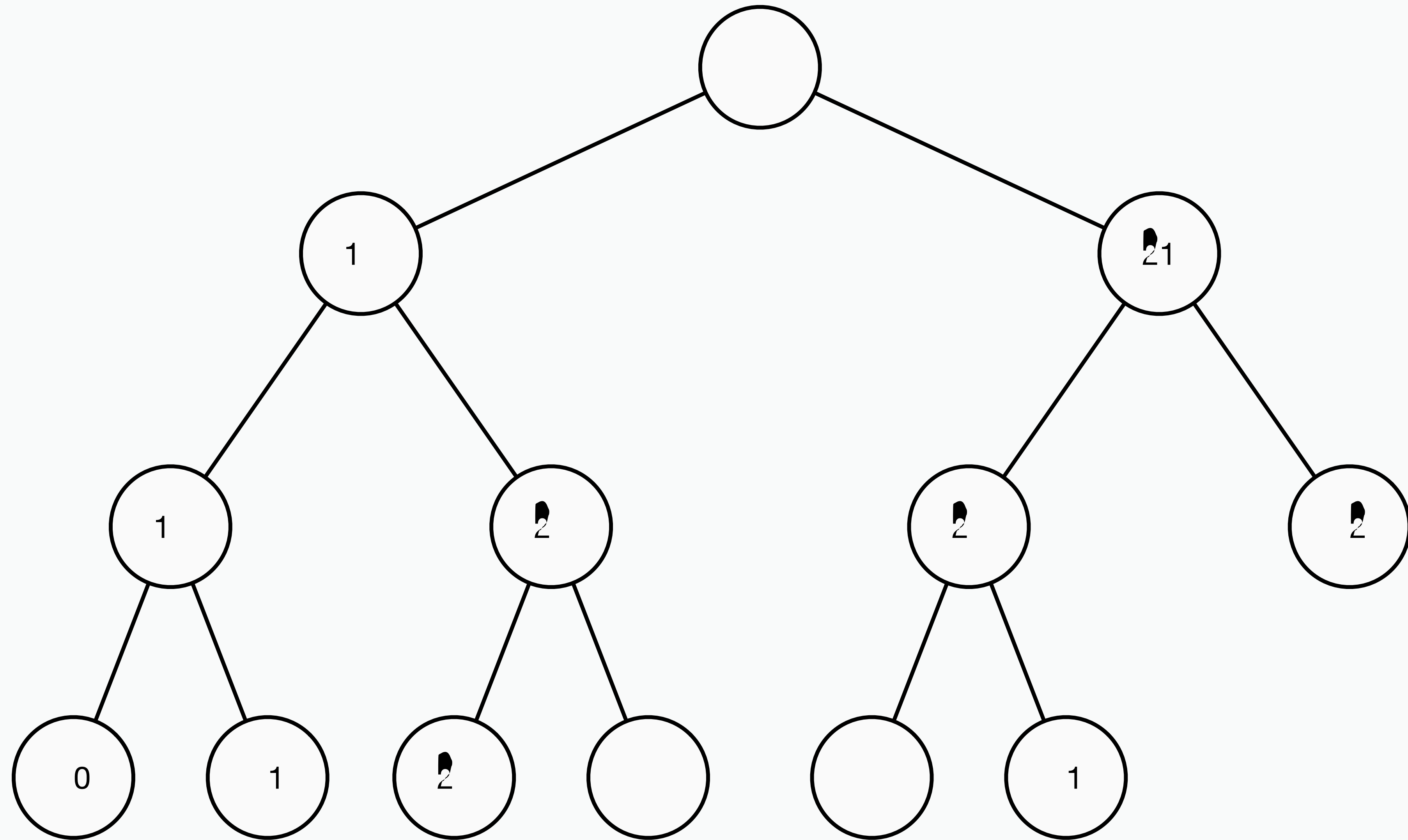
Graph algorithms (. . , shortest path, spanning tree)

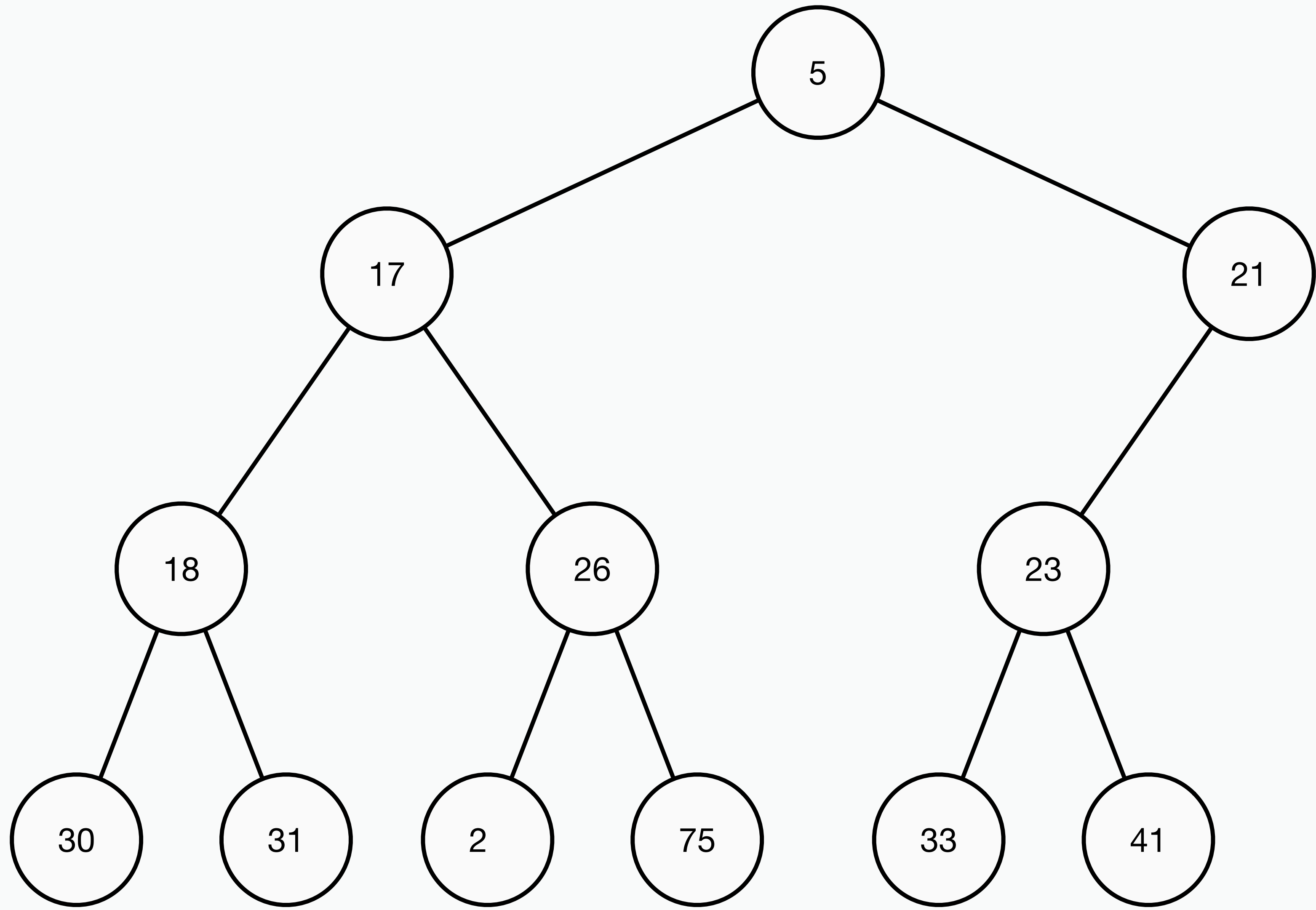
Priority queue (which itself has abundant applications)

It is important to keep in mind the distinction between a binary heap and a binary search tree (BST). They are **not** the same thing.

A binary search tree requires that values of a left subtree must be ordered with respect to the right subtree. This is **not** the case with binary heaps. The heap order property is a little more relaxed.

Binary heap **binary search tree**





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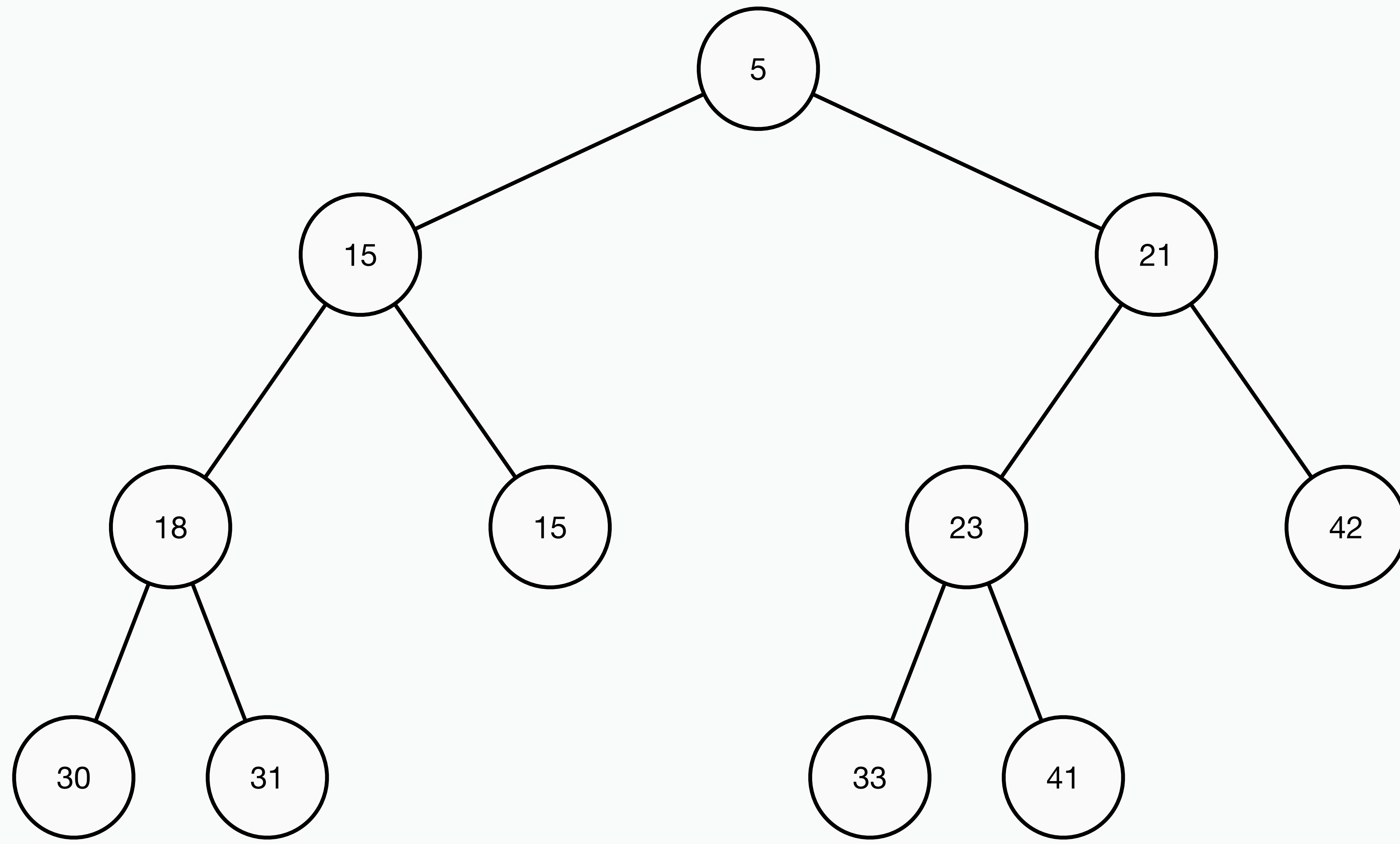
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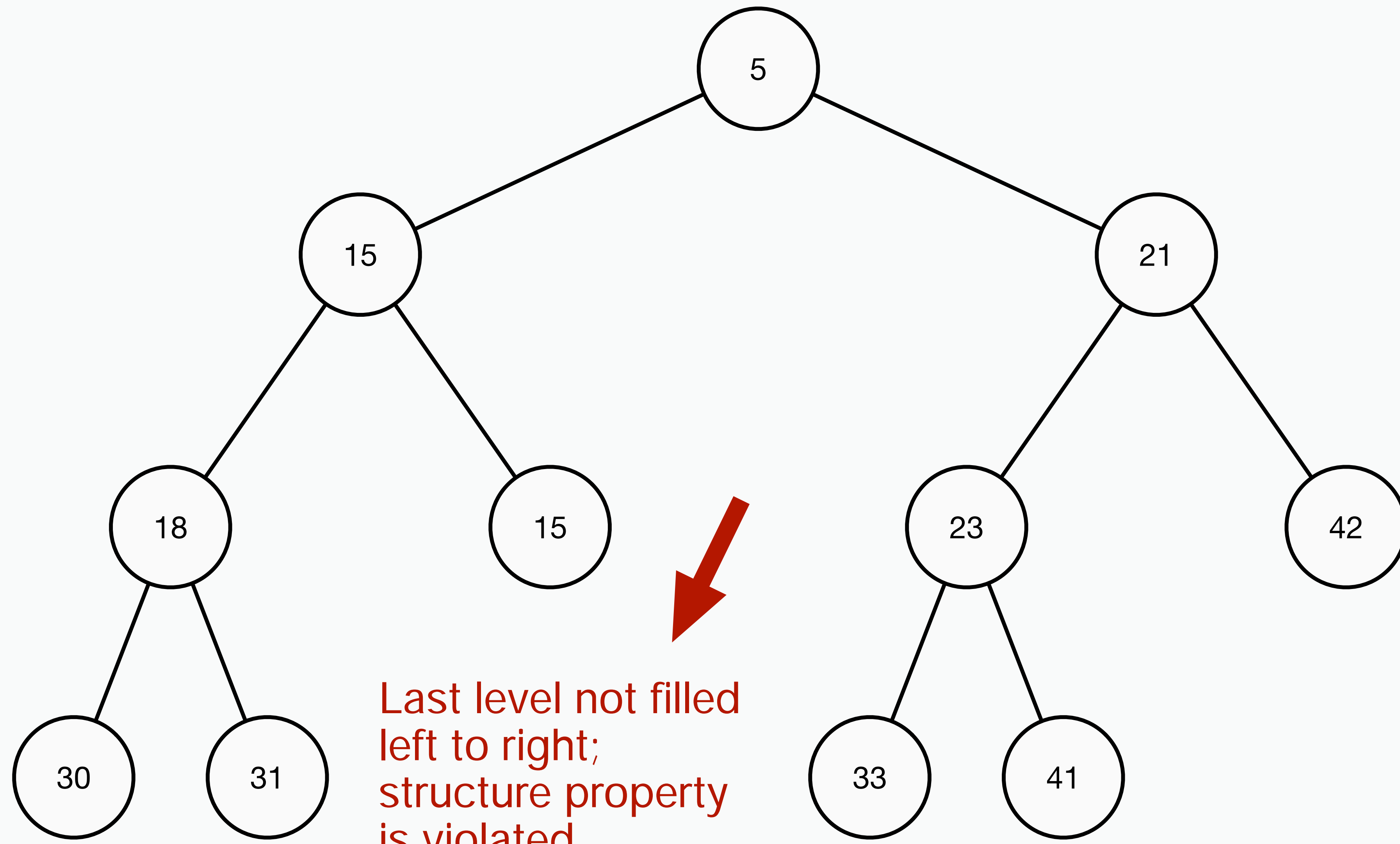
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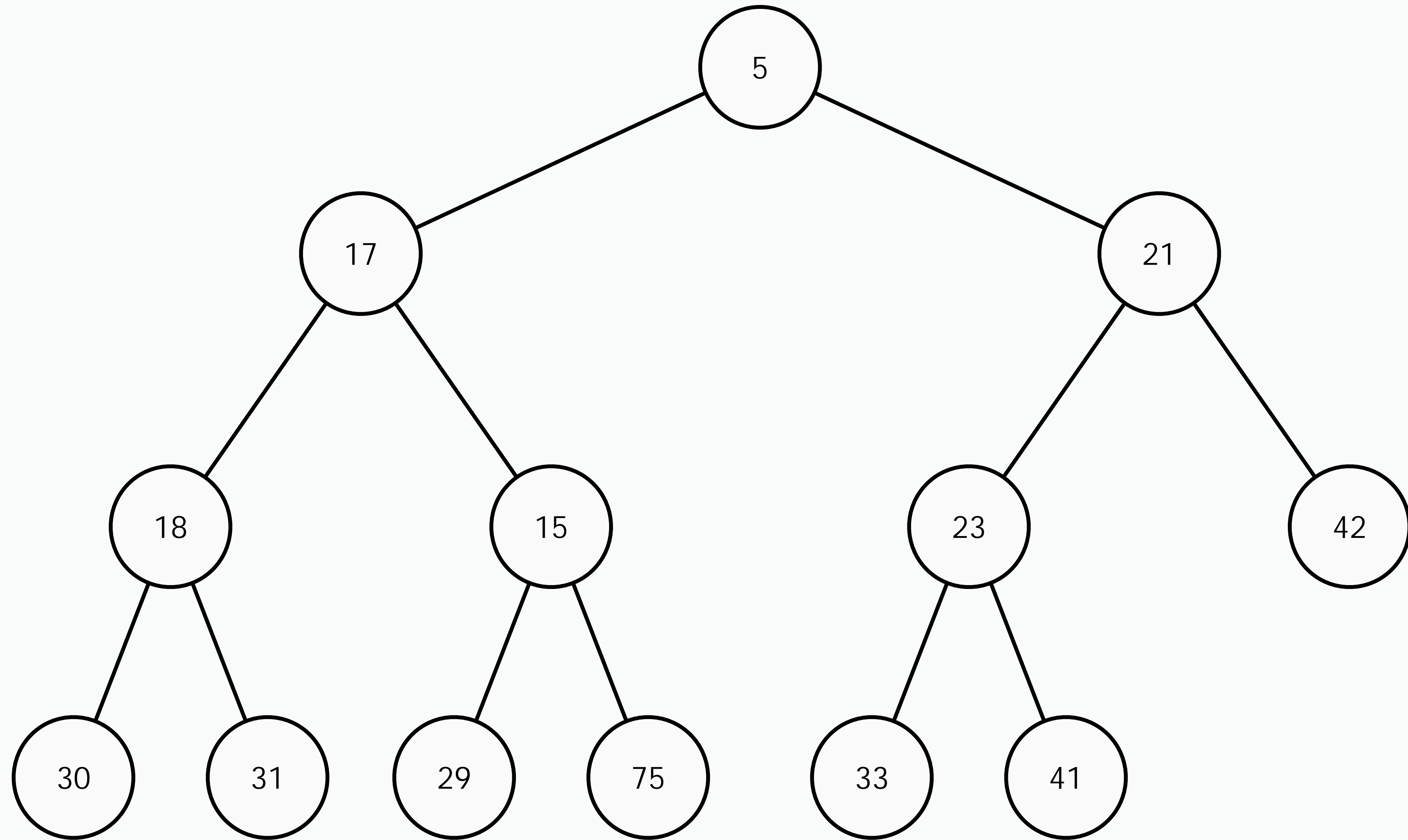
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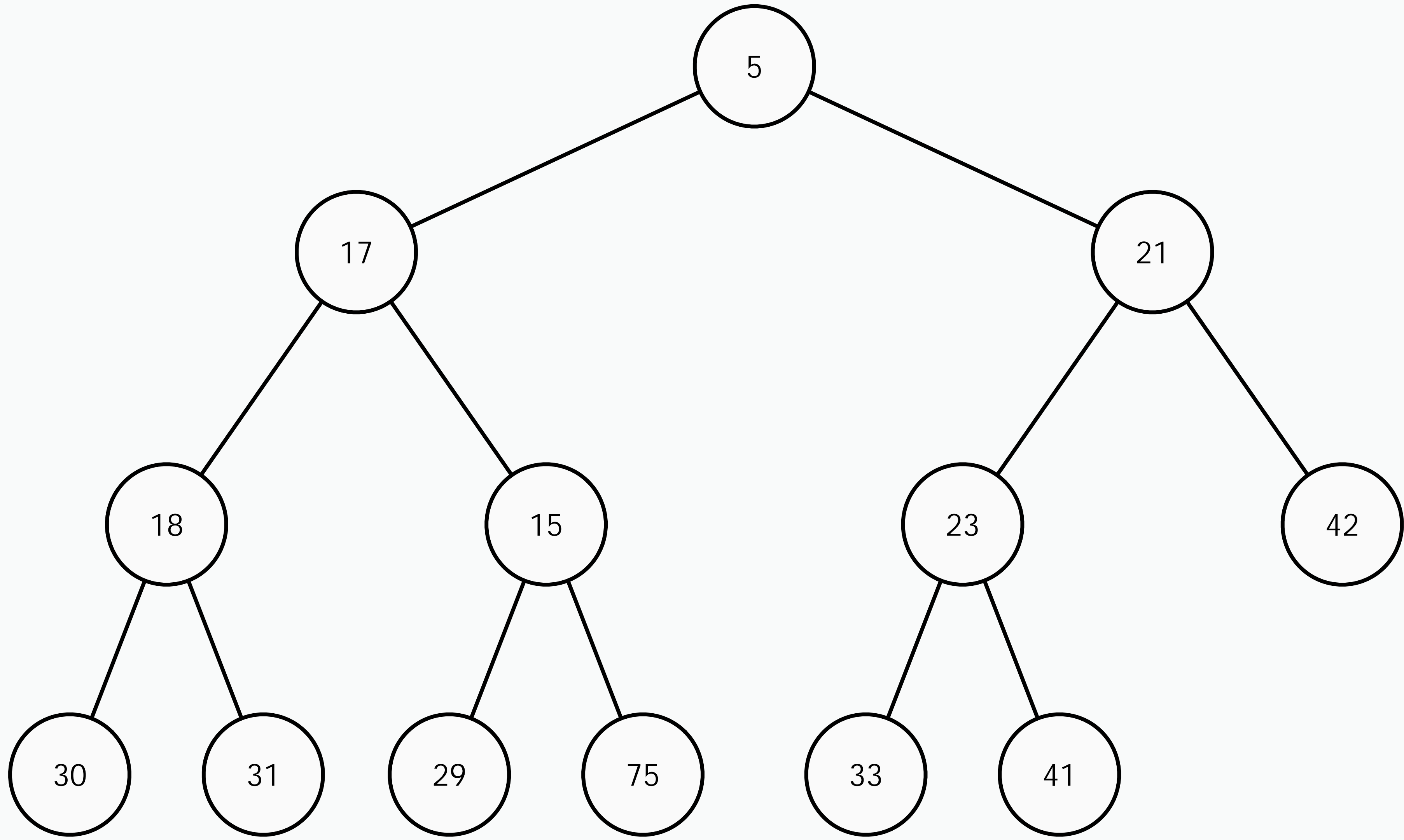
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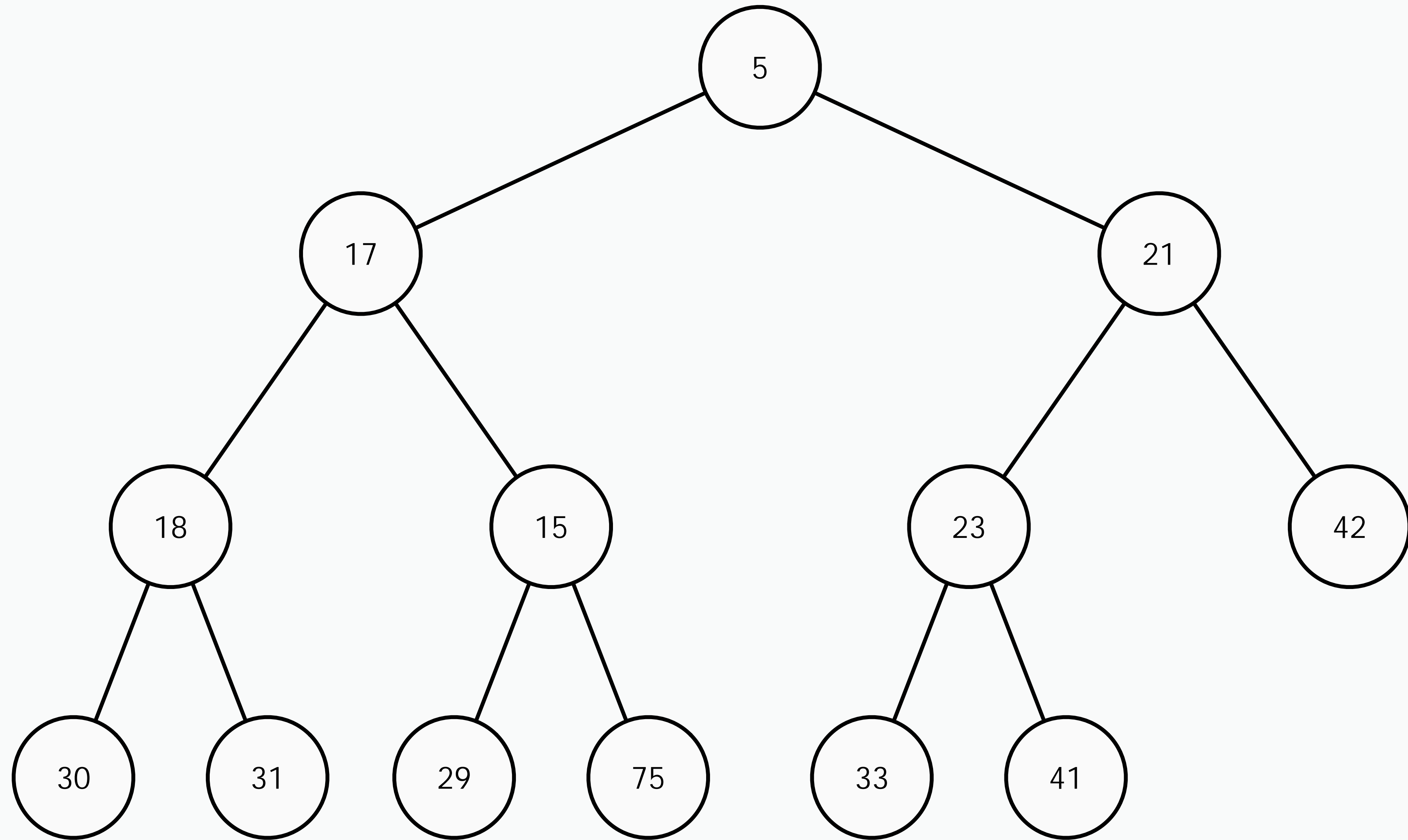




Last level not filled
left to right;
structure property
is violated.







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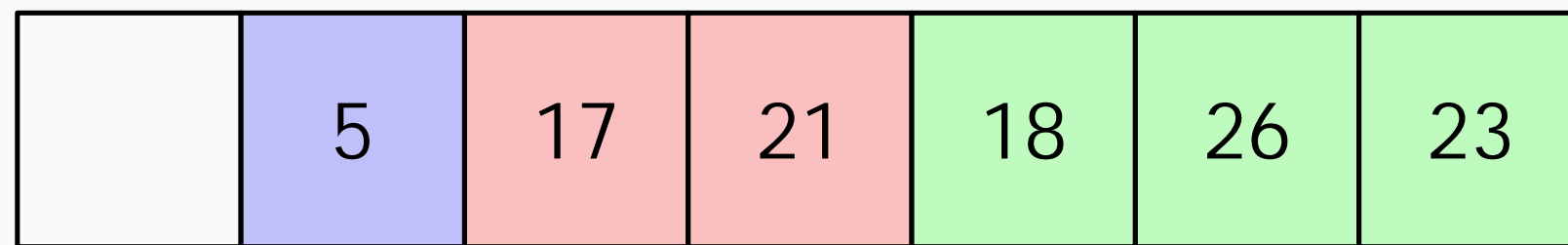
Recall that the height of a tree, h , is the length of the longest path from the root node to a leaf node.

Recall also that the structure property of a binary heap requires that the tree be a complete binary tree. A complete binary tree must have between 2^h and $2^{h+1} - 1$ nodes.

Given a tree of n nodes, its height is $\lfloor \log_2 n \rfloor$.

	5	17	21	18	26	23	42	30	31	
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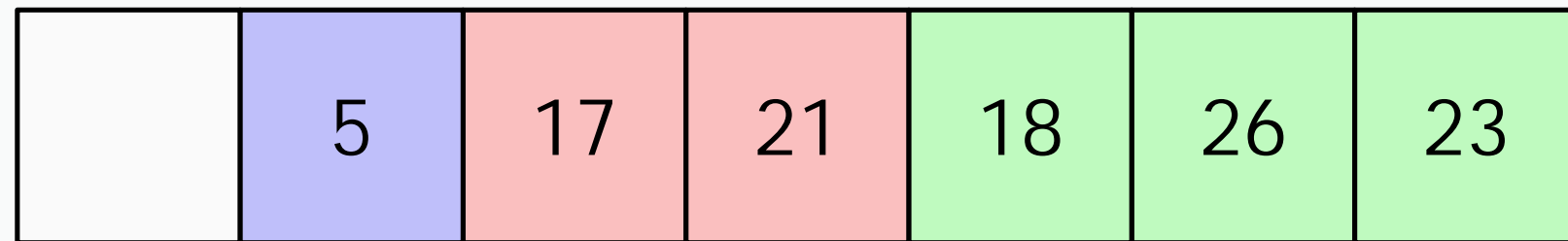
Because a binary heap has a highly regular structure, we can represent it with an array.



1 element in array

$2^0 = 1$ node in this level

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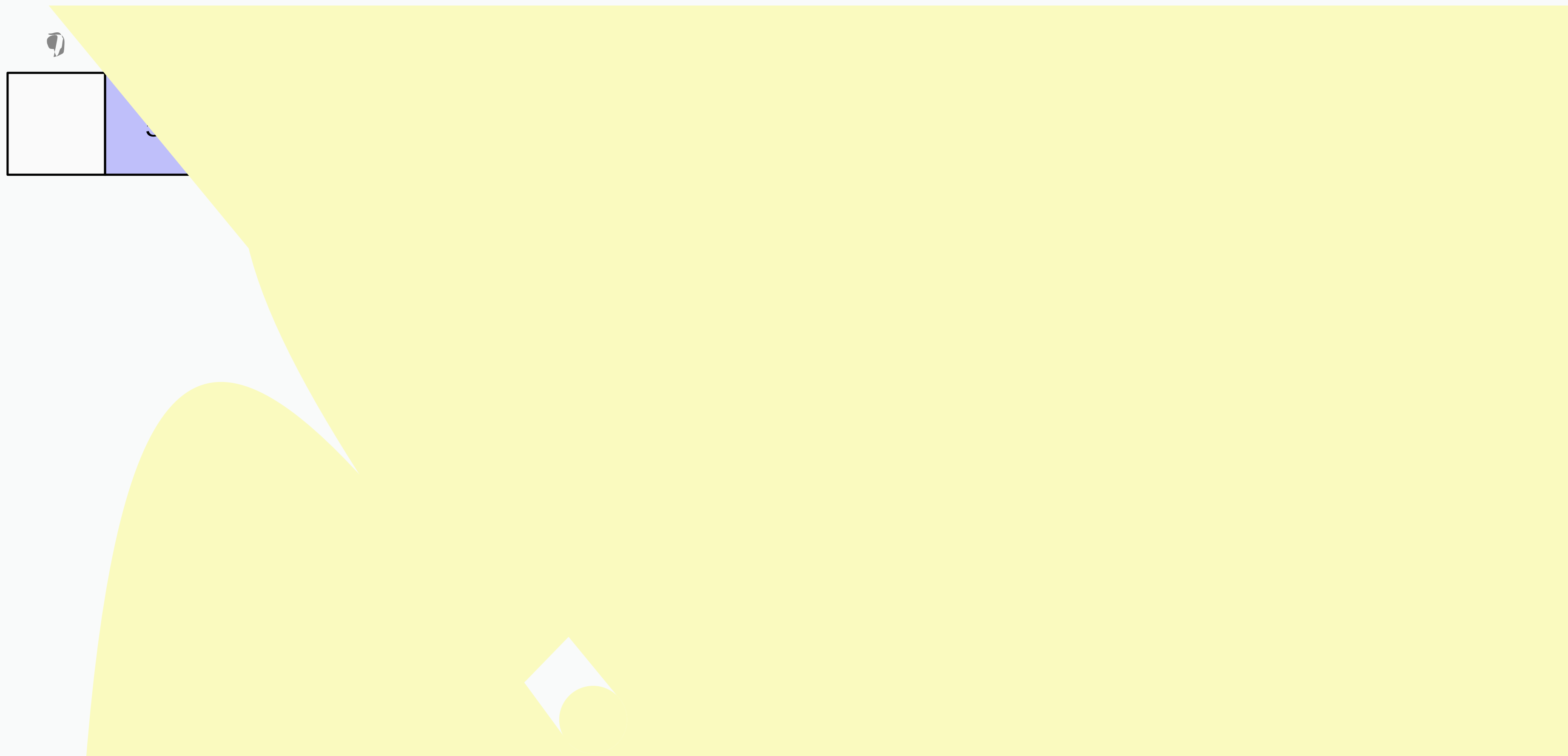


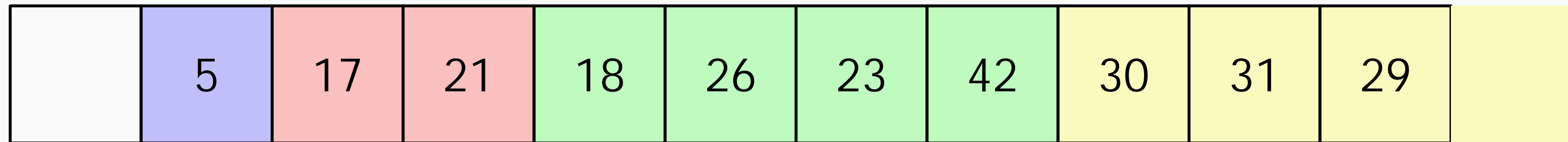
4 elements in array

$2^2 = 4$ nodes in this level

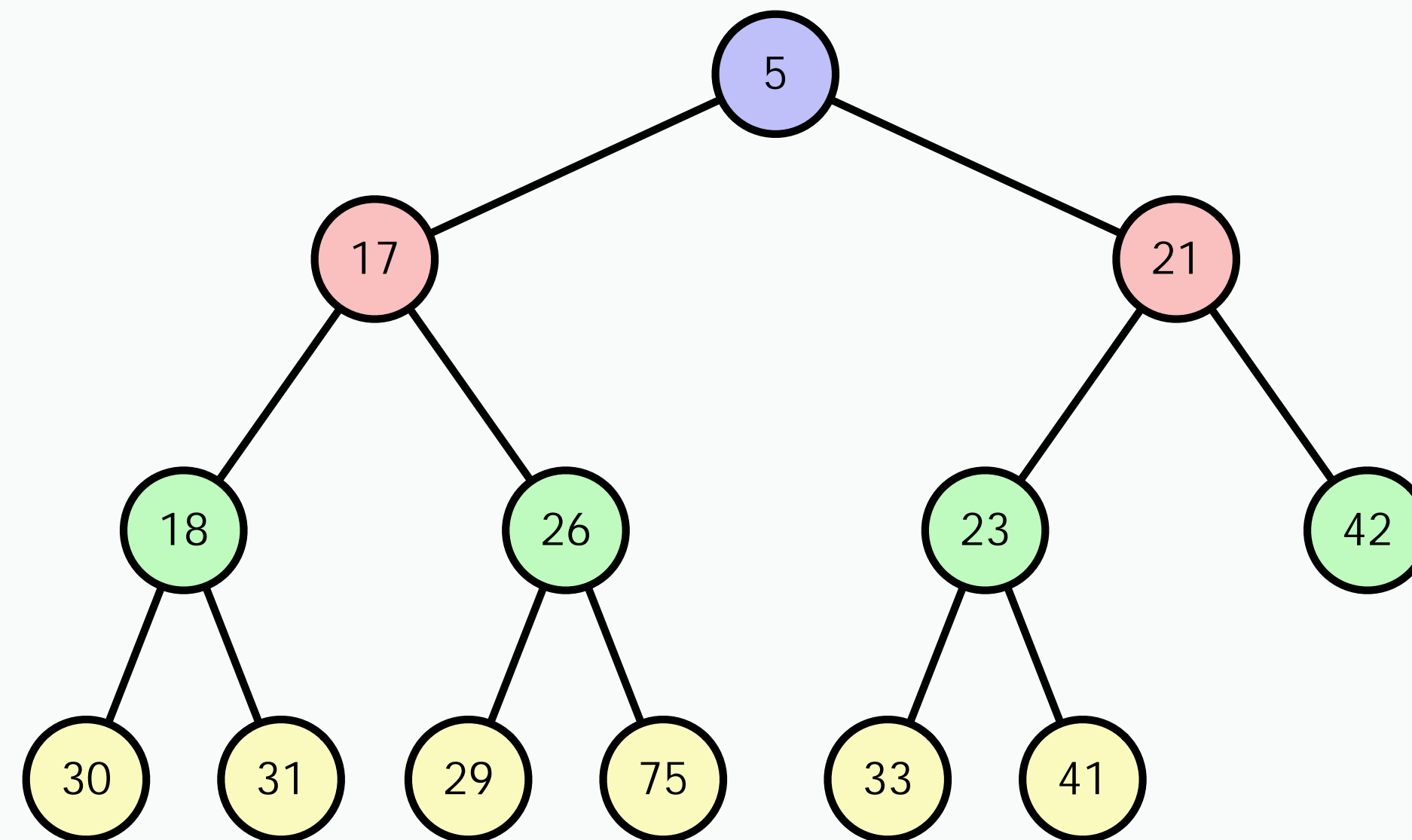
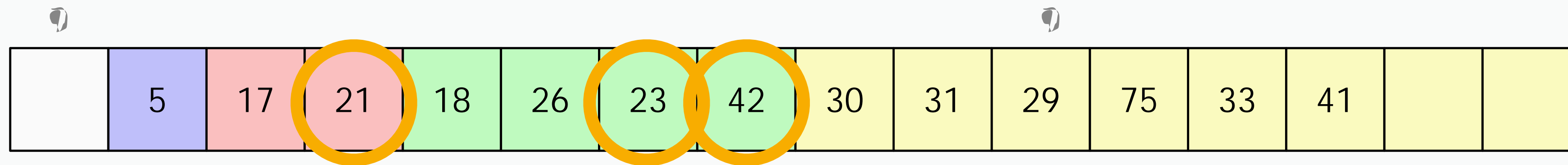
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Node at index 2 has children at indices 2 * 2 and 2 * 2 + 1.



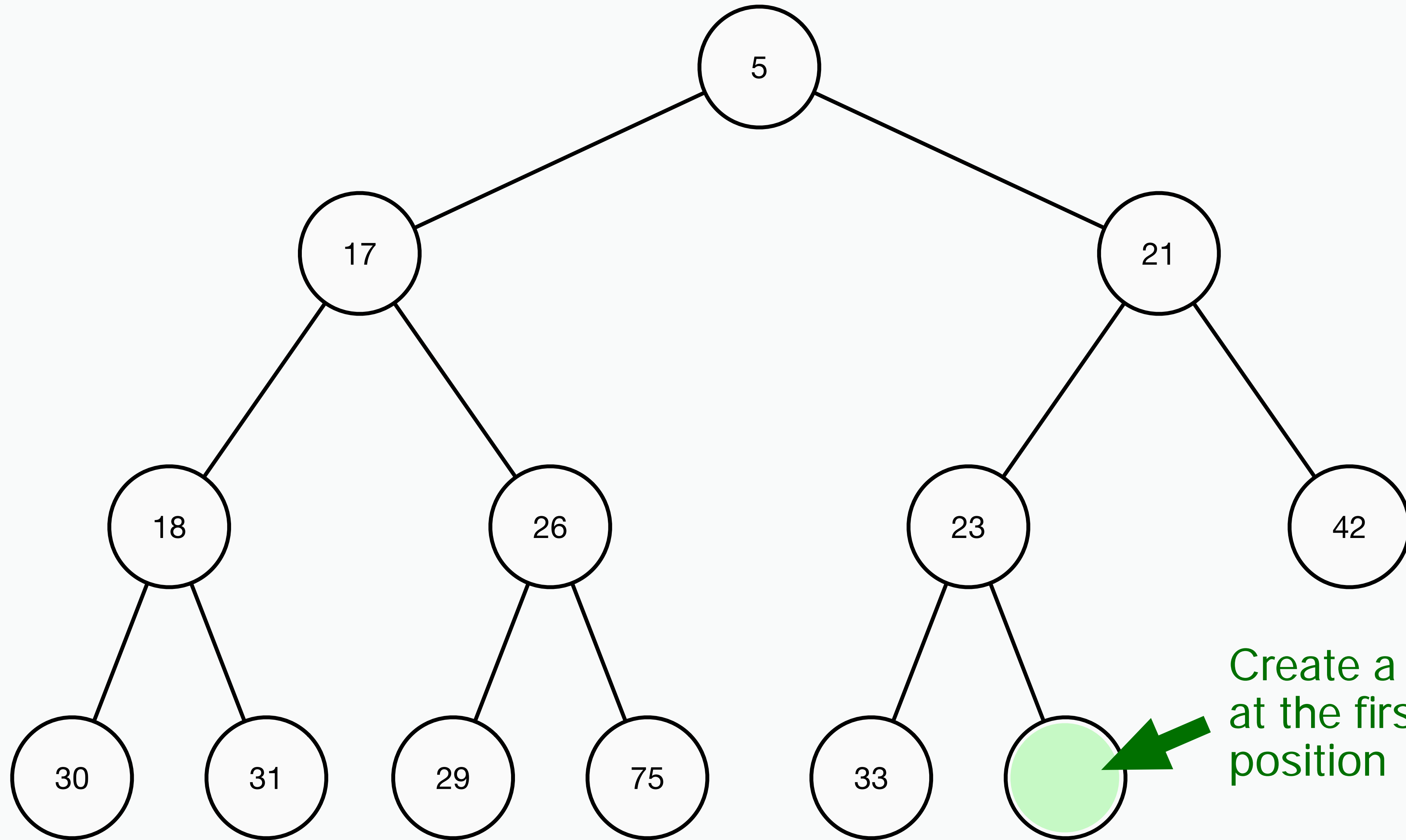
As a static object, a binary heap is of little utility. We need to be able to modify it by inserting and deleting nodes. But we must insert and delete in ways that

Let's look at the following operations:

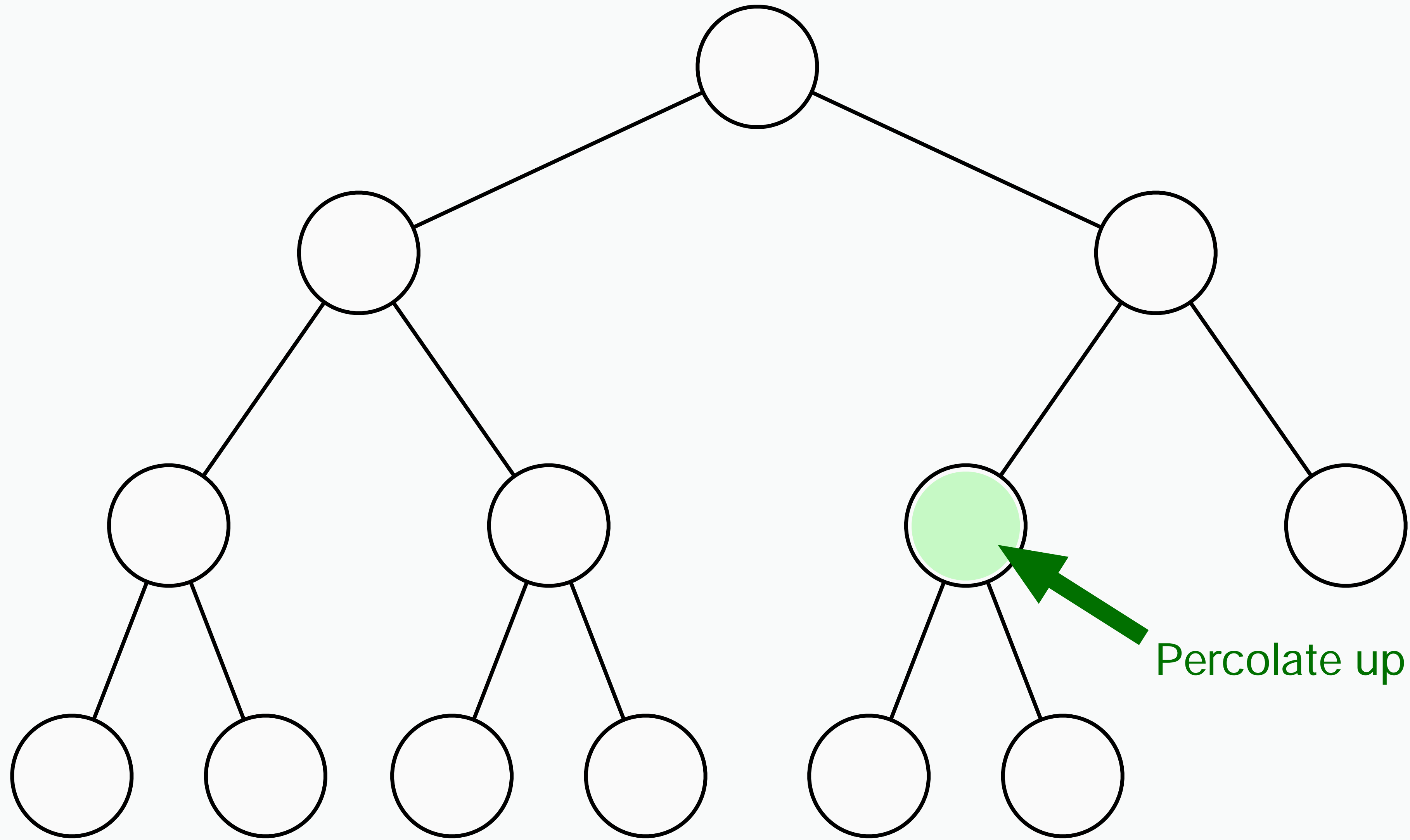
Insert

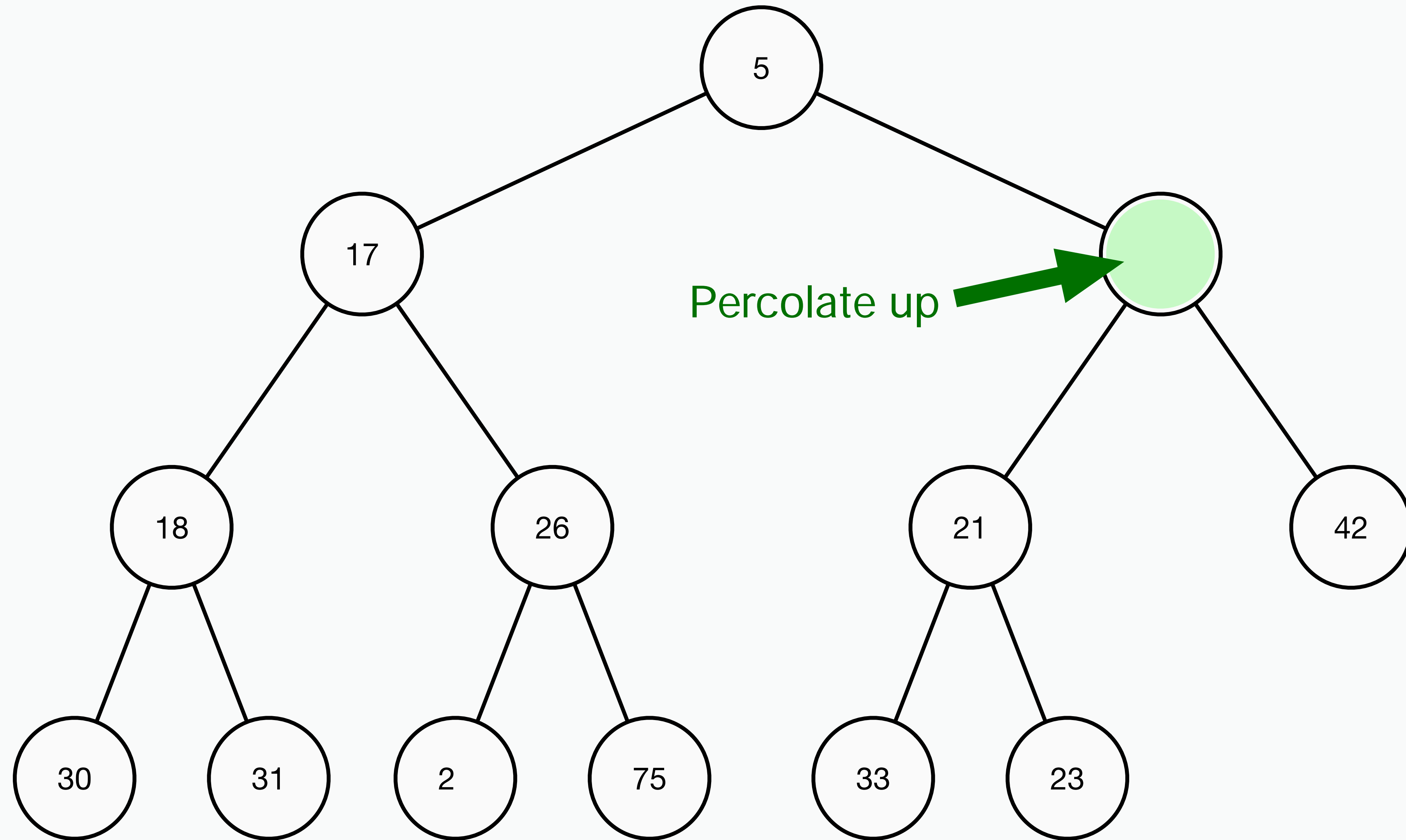
Delete minimum

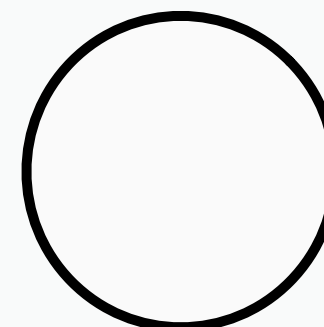
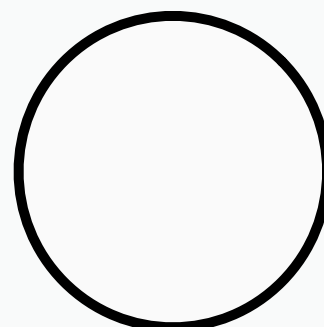
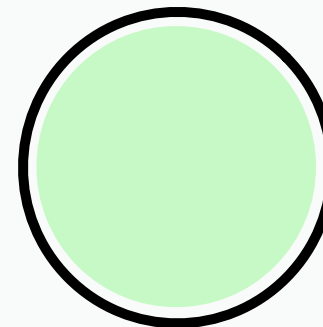
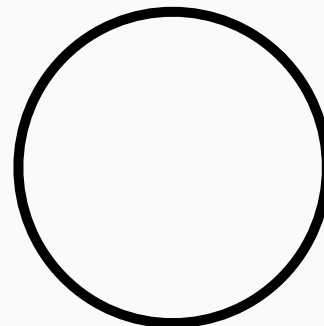
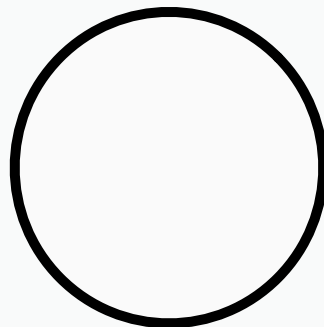
When inserting and deleting we use one of two strategies to preserve structure and heap-order property. We use the metaphor of a "bubble" or within the heap.

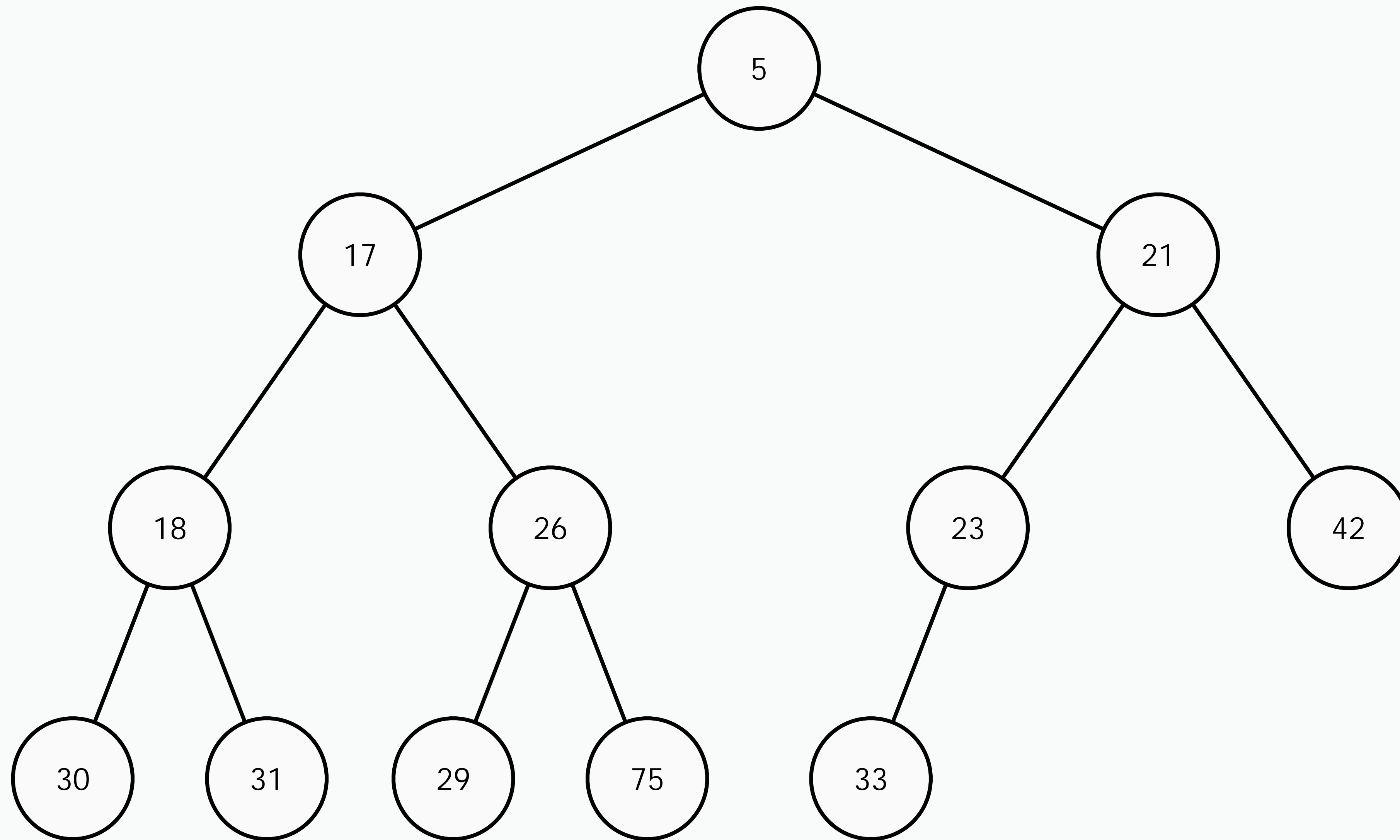


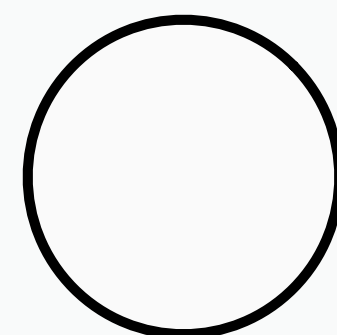
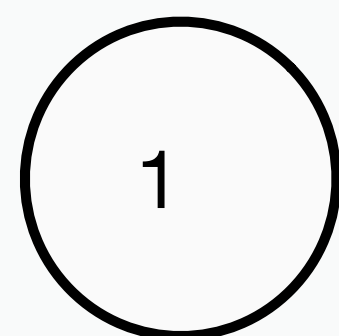
Create a "bubble"
at the first open
position

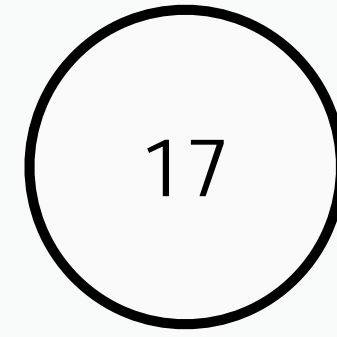
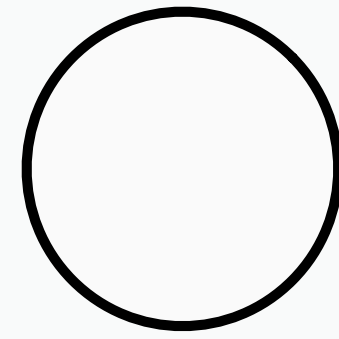


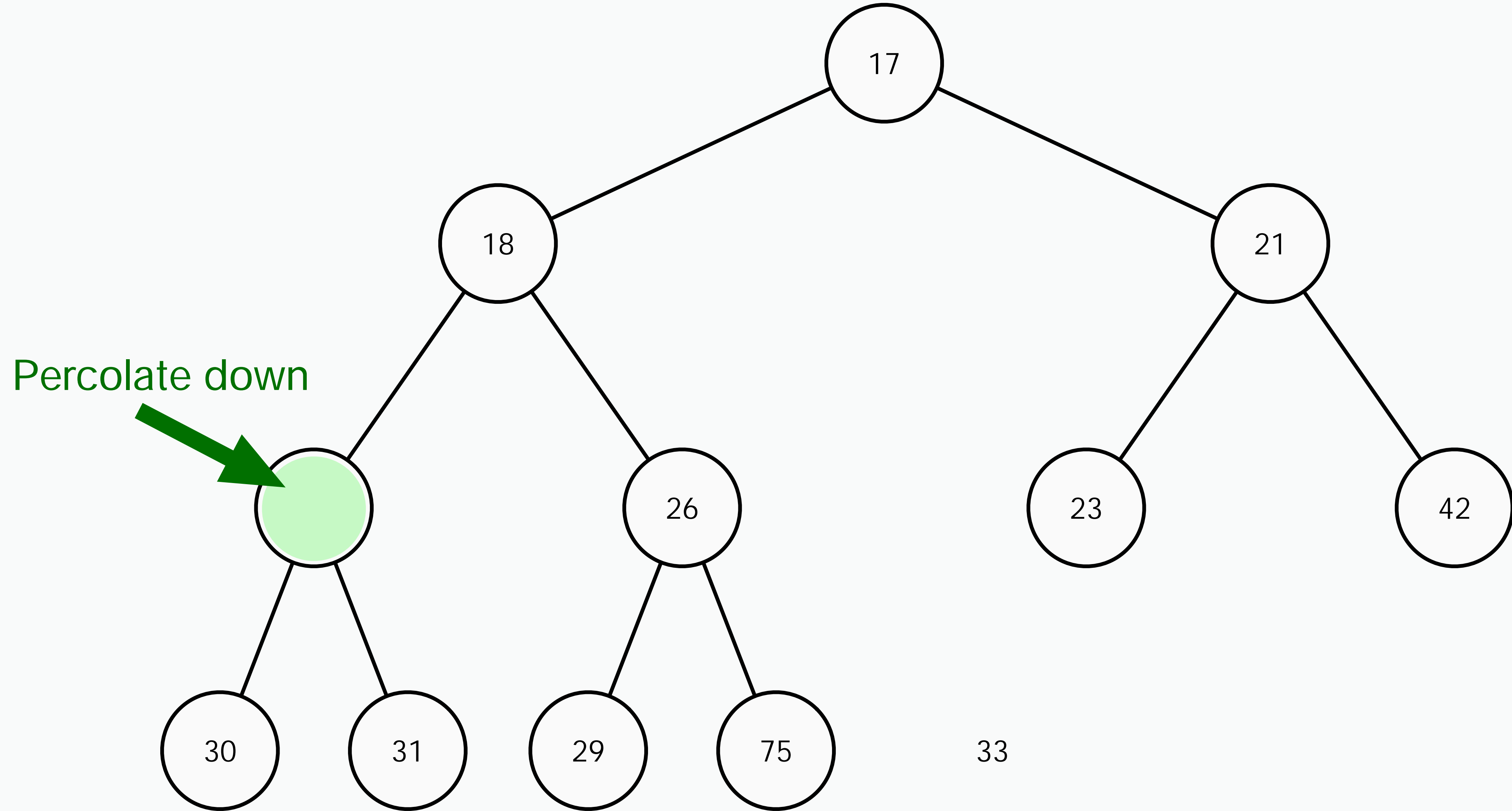








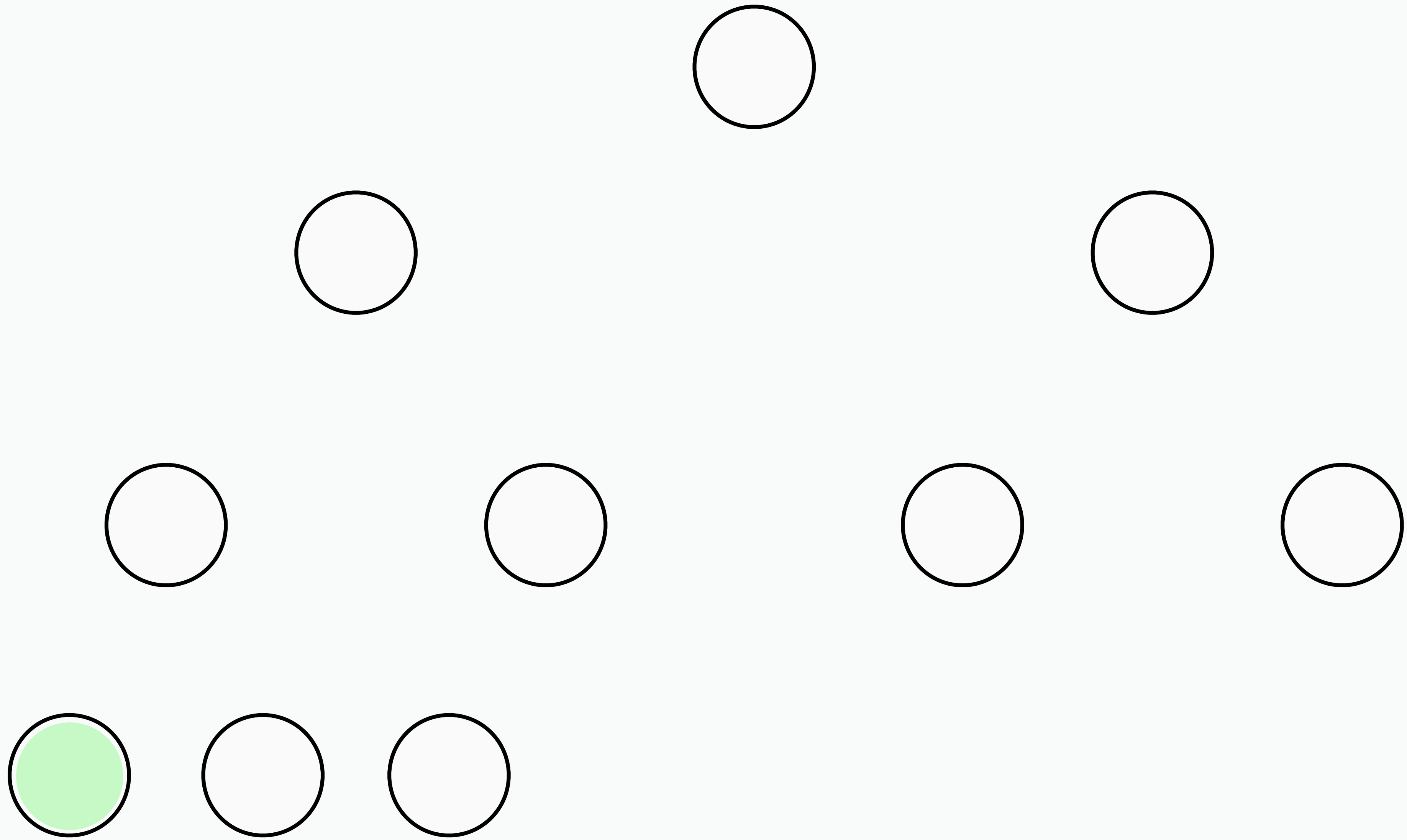


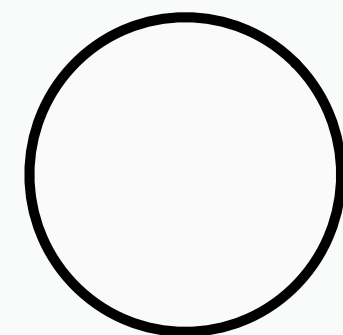
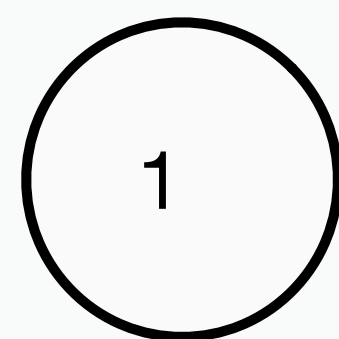


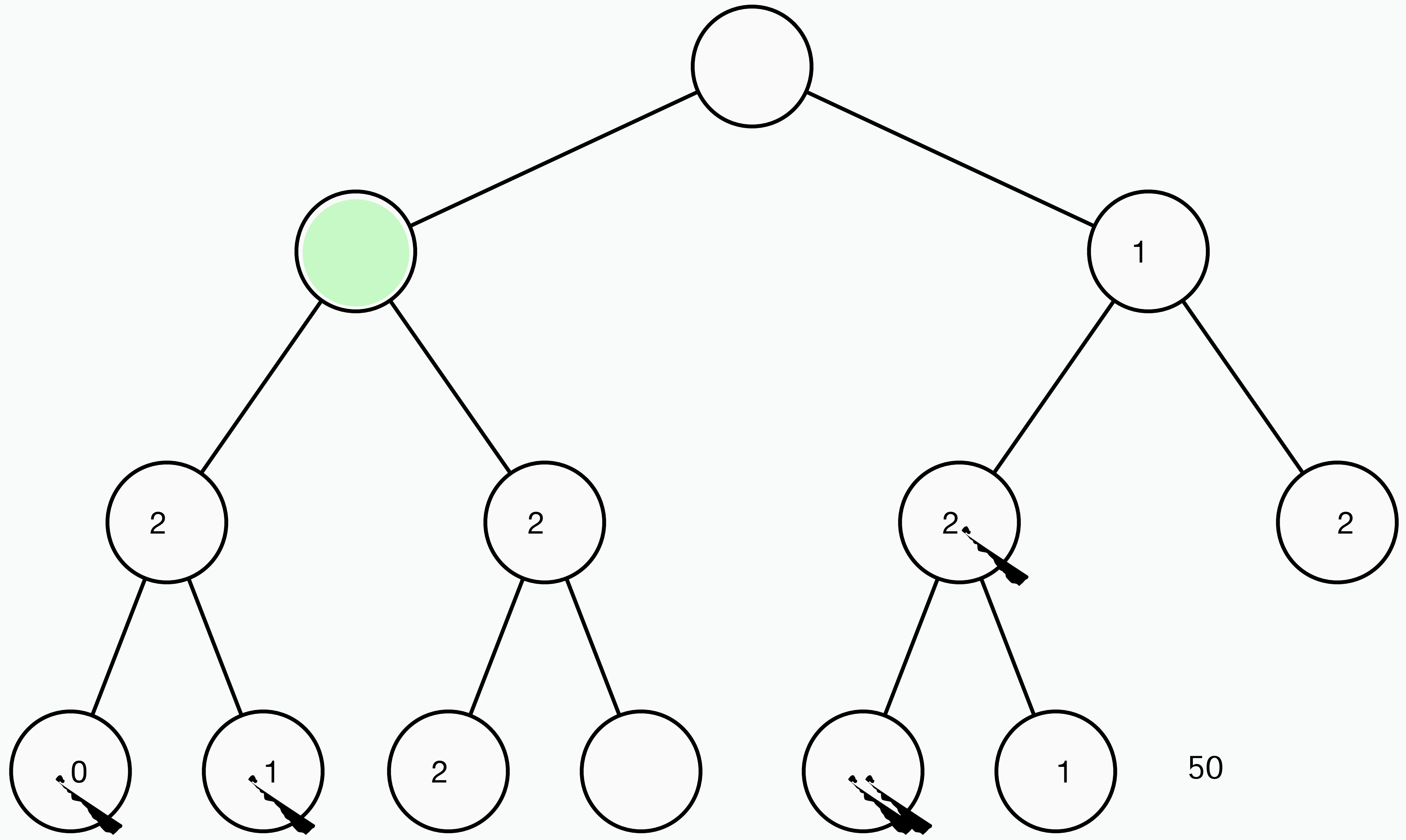
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Binary heap is a $-\log_2 n$ binary tree with the $-\log_2 n$ property.

Structure property and heap-order property must be preserved.

Binary heap can be represented with an array.

We modify the heap by creating $-\log_2 n$ and $-\log_2 n$ until a new (or orphaned) value can find a suitable node in the tree.

Insert and delete operations have $O(\log n)$ complexity.

Binary heaps are well-suited to $-\log_2 n$ and other applications.