REAL AND COMPLEX ANALYSIS PHD QUALIFYING EXAM

September 19, 2009

The test has two sections, covering real and complex analysis. In order to pass, you must do at least 2 problems from each section **completely correctly**, and you must do a total of 6 problems completely correctly, or 5 completely correctly with substantial progress on 2 others. Some problems have more than one part (e.g., problem 1 in Section I consists of 1a), 1b), and 1c)).

I. REAL ANALYSIS.

1. Let $(X; d)$ be a metric space. Show that, if $\{x_n\}$ is a sequence in X and $p \in X$, then $x_n \to p$ if and only if every subsequence from $\{x_n\}$ has itself a subsequence that converges to p.

2a) Suppose that $f : \mathbb{R} \mapsto \mathbb{R}$ is differentiable everywhere, and that

$$\lim_{x \to \infty} f'(x) = 0.$$

Show that

$$\lim_{x \to \infty} \frac{f(x)}{x} = 0.$$

2b) Use 2a) to prove the following: If $f : \mathbb{R} \mapsto \mathbb{R}$ is differentiable everywhere, and

$$\lim_{x \to \infty} f'(x) =$$

$P_0 = \{(0; 0; 0)\},\ P$

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