Numerical Analysis PhD Qualifying Exam

May 2020

Instructions:
Four problems must be completed, and one problem must be attempted. At least two problems from 1{4 (group 1) and at least two problems from 4{7 (group 2) must be completed. Note that Problem 4 can count towards either group, but not both. To have attempted a problem, you must correctly outline the main idea of the solution and begin the calculation, but need not nish it. You have three hours to complete the exam.

1. For the equation $f(x) = 0$, suppose x is a triple root, i.e. $f(x) = f'(x) = f''(x) = 0$ but $f'''(x) \neq 0$. If one uses Newton's method
$$x_{n+1} = x_n \quad \frac{f(x_n)}{f'(x_n)}$$
to compute this root starting from an initial condition x_0 that is close to this root, will this method converge? If not, explain why. If yes, determine its convergence rate.

2. Compute the rst three steps of the Jacobi and Gauss{Seidel iterations with starting vector $[1; 0; 1]^T$ by hand for the following linear system of equations:
$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

3. Consider the following numerical formula to approximate the integral
$$\int_0^1 x f(x)\,dx \quad a_1 f(0) + a_2 f\left(\frac{1}{2}\right) + a_3 f(1);$$
where $f(x)$ is a smooth function. Determine the coe cients $a_1; a_2$ and a_3 so that this formul1452(this)-45

6. Consider Problem 5 above, where it is given that $p(x) > 0$ for x 2 $[0; 1]$. Explain how you would solve this boundary-value problem by the shooting method (with an arbitrary h, of course). You should describe and explain all relevant details, but should not solve any equations.

7. Consider a unidirectional wave equation

$$u_t = c\, u_x; \qquad 1 < x < 1 \tag{2}$$

where c = const.

Use the von Neumann analysis to determine if there is any range of parameter

$$= \frac{c}{h} > 0$$

for which the scheme

$$\frac{U_m^{n+1} \quad U_m^n}{} = c\,\frac{U_m^n \quad U_{m \quad 1}^n}{h};$$

approximating (2), would be stable.