Differential Equations PhD Qualifying Exam
University of Vermont
January 11, 2017

Name: ______________________________

- Time allowed: 3 hours.
- Brains only: No calculators or other electronic gadgets allowed.
- **Two problems from each section must be completed correctly, and one additional problem from each section must be attempted**. In an attempted problem, you must correctly outline the main idea of the solution and start the calculations, but do not need to finish them. **Numerical criteria for passing**: A problem is considered completed (attempted) if a grade for it is $\geq$ 85% ($\geq$ 60%).

Section 1, ODE

1. Draw the phase portrait for the system

$$\dot{x} = x(2 - x - y)$$
$$\dot{y} = x - y$$

and identify the fixed points and their stability.

2. Solve the non-homogeneous linear system

$$\dot{\mathbf{x}} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} t \\ 1 \end{pmatrix}$$

with the initial condition $\mathbf{x}(0) = [1\ 0]^\top$.

3. Express the linear system of ODEs

$$\dot{x}_1 = a x_1 - b x_2$$
$$\dot{x}_2 = b x_1 + a x_2$$

in polar coordinates, where $r^2 = x_1^2 + x_2^2$ and $\theta = \tan^{-1}(x_2/x_1)$. The result should have a very simple form. Then solve using the initial conditions $r(0) = r_0;\ \theta(0) = \theta_0$.

4. Consider the biased van der Pol oscillator $\ddot{x} + \mu(x^2 - 1)\dot{x} + x = a$. Find the curves in $(\mu; a)$ space at which Hopf bifurcations occur.

5.