

COMBINATORICS QUALIFYING EXAM

August 2021

You have four hours to complete this exam.

When working on later parts of a problem, you may assume the results of earlier parts of the same problem without proof.

PhD Pass: Three numbered questions solved completely, or two questions solved completely and substantial progress on two additional questions.

MS Pass: Substantial progress on three questions.

Section A

Question 1

Let G be a simple, undirected, connected graph. For two vertices $a, b \in V(G)$, we let $d(a; b)$ denote the distance between them, i.e. the length of a shortest path in G . We let $D(a; b)$ denote the length of a longest path in G .

(a) Prove that, for distinct vertices $a, b, c \in V(G)$,

$$d(a; b) + d(b; c) = d(a; c);$$

(b) Prove that, for distinct vertices $a, b, c \in V(G)$, $D(a; b) + D(b; c) = D(a; c)$. We let $D(a; b)$

