ALGEBRA PH.D. QUALIFYING EXAM

January 10, 2014

A passing paper consists of four problems solved completely plus significant progress on two other problems; moreover, the set of problems solved completely must include one from each of Sections A, B and C.

## Section A.

In this section you may quote without proof basic theorems and classifications from group theory as long as you state clearly what facts you are using.

1. Let G be a group of order 3393 (note that $3393 = 3^2 \cdot 13 \cdot 29$).

    (a) Compute the number, $n_p$, of Sylow p-subgroups permitted by Sylow's Theorem for each of $p = 3$, 13, and 29.

    (b) Show that G contains either a normal Sylow 13-subgroup or a normal Sylow 29-subgroup.

    (c) Show that G

5. Let $R$ be a Principal Ideal Domain, let $M$ be an $R$-module, and let $p$ be a nonzero prime in $R$.
   Define

$$M_p = \{m \in M \mid p^a\}$$