Two problems from each Section must be completed, and one additional problem from each Section must be attempted. In an attempted problem, you must correctly outline the main idea of the solution and start the calculations, but do not need to finish them. Numeric criteria for passing: A problem is considered completed (attempted) if a grade for it is $\geq 85\%$ ($\geq 60\%$).

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1. Sketch a phase portrait of the equation

5. Solve the following wave equation on the semi-in nite domain:

$$u_{tt} = u_{xx}; \qquad 0 \leq x < \infty; \quad t \geq 0;$$

$$u(x;0) = f(x); \quad u_t(x;0) = g(x); \quad x \geq 0;$$

$$u_x(0;t) = 0; \quad t \geq 0:$$

6. For $0 \leq x \leq$, solve the following heat equation with a source and initial-boundary conditions:

$$\phi_t = \phi_{xx} + w(x;t);$$

$$\phi(0;t) = 0; \quad \phi_x(\ ;t) = 0;$$

$$\phi(x;0) = f(x):$$

7. Use the method of characteristics to solve the problem:

$$\begin{aligned} u_t + uu_x &= 0; \quad -\infty < x < \infty; \quad t > 0 \\ u(x;0) &= e^{-x^2}; \quad -\infty < x < \infty; \end{aligned}$$

and express your solution in terms of the initial-condition function (perhaps implicitly). In particular, describe how you would evaluate your solution at, say, $x = 1$ and $t = 0{:}1$. In the $(x;t)$-plane, sketch several typical characteristics. When and where will shocks form?

8. Consider the following eigenvalue problem

$$\phi'' - x\phi + x^2\phi = 0; \quad 0 \leq x \leq 1;$$

$$\phi(0) = 0; \quad \phi'(1) + \phi(1) = 0:$$

(1) Show that its eigenvalues are real;
(2) Show that eigenfunctions corresponding to di erent eigenvalues are orthogonal under a certain weighting function.