REAL AND COMPLEX VARIABLES PH.D. QUALIFYING EXAM

September 17, 2015

Three Hours

A passing paper consists of a total of six problems done completely correctly, or five problems done correctly with substantial progress on two others. At least three problems from each of Section A (Real Analysis) and Section B (Complex Analysis) must count toward the passing criteria, and two of these from each section must be completely correct.

Section A. Real Analysis

1. Let ( ) be a metric space, and let $\{x_n\}_{n=1}^{\infty}$ and $\{\ _n\}_{n=1}^{\infty}$ be Cauchy sequences in .
   Prove that the sequence of real numbers $\{\ (x_n\ \ _n)\}_{n=1}^{\infty}$ converges in $\mathbb{R}$.
   (Do not assume is complete.)

2. Let $_n : \mathbb{R} \to \mathbb{R}$ be a sequence of functions that converges uniformly to a function $: \mathbb{R} \to \mathbb{R}$. Prove that if the sequence of real numbers $\{a_n\}_{n=1}^{\infty}$ converges to a and is continuous at a, then the sequence $\{\ _n(a_n)\}_{n=1}^{\infty}$ converges to (a).

3871(t)484o)-5.19.74v1 -448.588 -14.28 Td [(P)-0.5316976(n)6v1'646.459 -34.32 2 7.6608 Tf=209 0 0 1 1 0 Td (a)R13 10.9655 Tf 7

-3.12 e Tf 5.42055 0 Td [(=)-42351(A)1.669-3.632)-3.08g877(A)1.60(r)-3.80569(g)5.6616(e)5.03130-357 -45156(i)3.115 T240(e)5+ b.

Section B. Complex Analysis

8. Identify explicitly the real and imaginary parts of the function ( ) = cos , and verify any **one** of the Cauchy–Riemann equations for at an arbitrary point