Numerical Analysis: MATH 337

Qualifying Examination

January 10, 2018

1. The Dahlquist method

$$Y_{n+1} - 2Y_n + Y_{n-1} = \frac{h^2}{4}(f_{n+1} + 2f_n + f_{n-1}), \qquad \text{where } f_n \equiv f(x_n, Y_n), \text{ etc.} \qquad (1)$$

can be used to solve the second-order initial-value problem

$$y'' = f(x, y), \qquad y(x_0) = y_0, \quad y'(x_0) = y_0'. \qquad (2)$$

Show that method (1) has the global error of order 2 when applied to (2).

2. Consider the boundary-value problem

$$u''(x) + 13u'(x) + 17u(x) = R(x), \qquad u'(0) = \ , \qquad u(1) = \ ,$$

where $R(x)$ is an arbitrary given function and and are arbitrary given constants. Note the Neumann boundary condition at the left end point.

Write down the equations of a method that would approximate the solution of this problem with accuracy $O(h^2)$. You may choose any of the methods considered in the course MATH 337. Provide all necessary explanations.

3. Consider a unidirectional wave equation

$$u_t = c\,u_x, \qquad -1 < x < 1 \qquad (1)$$

where c = const.