COMPLEX VARIABLES PH.D. QUALIFYING EXAM

September 27, 2008

There are ten questions. A passing paper consists of seven problems done completely correctly, or six problems done correctly with substantial progress on two others. Let $\mathbb{D}$ denote the open disc of radius 1 centered at the origin.

1. Let $f(z) = z|z|^2$.

(a) Find all points in the complex plane where f satisfies the Cauchy-Riemann equations.

(b) Does f have a complex derivative at the points you found in (a)? (Justify briefly.)

2. Find all complex numbers z such that $\tan z = i - 1$.

3. Let $f(z) = \bar{z}$. In each of parts (a) and (b) compute $\int f(z)\,dz$, where is the specified path whose initial point is -1 and terminal point is i.

(a) is the path along the coordinate axes: from -1 to 0 and then from 0 to i.

(b) is the quarter of the unit circle lying in the second quadrant, oriented clockwise.

(c) Could there be a function g that is analytic on some simply connected open set U containing both the paths in (a) and (b) such that [illegible]

7. **Let f and g be analytic on the closed unit disc $\overline{\mathbb{D}}$, and assume both f and g have no zeros in $\overline{\mathbb{D}}$. Prove that if $|f(z)| = |g(z)|$ for all z with $|z| = 1$, then $f(z) = kg(z)$ in $\mathbb{D}$ for some constant k of modulus 1.**

8. (a) **Exhibit a function f such that at each positive integer n, f has a pole of order n, and f is analytic and nonzero at every other complex number. (Brie y justify your answer.)**

(b) **Let f be any function that satis es the conditions of part (a). For each positive integer N nd $\int_{C_N} \frac{f'(z)}{f(z)}\, dz$, where C_N is the circle of radius $N + \frac{1}{}$**