

## COMPLEX VARIABLES PH.D. QUALIFYING EXAM

September 28, 2007

*There are ten questions. A passing paper consists of seven problems done completely correctly, or six problems done correctly with substantial progress on two others.*

1. Find all solutions (if any) to the equation  $i^z = 2$ , where  $i = \sqrt{-1}$ .
2. Let  $C_1$  and  $C_2$  be the circles centered at the origin in  $\mathbb{C}$ , of radii 1 and 2 respectively. Evaluate, with brief justifications, the integrals,  
(a)  $\int_{C_1} \frac{e^z}{z-i} dz$                       (b)  $\int_{C_2} \frac{e^z}{z-i} dz$ .

3. Use the calculus of residues to evaluate the improper integral  
 $\int_{-\infty}^{\infty} \frac{1}{(x^2 + 1)^2} dx$ :

4. Find each singularity in  $\mathbb{C}$  and classify it (removable, pole of order  $n$ , essential) for the function  $f(z) = \frac{1}{z^2 + 1}$ .