ALGEBRA PH.D. QUALIFYING EXAM

May 20, 2008

A passing paper consists of four problems solved completely plus signi cant progress on two other problems; moreover, the set of problems solved completely must include one from each of Sections A, B and C.

Section A.

In this section you may quote without proof basic theorems and classi cations from group theory and group actions as long as you state clearly what facts you are using.

1. **Let G be a solvable group of order $168 = 2^3 \cdot 3 \cdot 7$. The aim of this exercise is to show that G has a normal Sylow p-subgroup for some prime p. Let M be a minimal normal subgroup of G.**

5. Let R be an integral domain and assume R contains a subring F that is a eld (R and F have the same 1). Prove that if R is nite dimensional as a vector space over F then R is a eld.

6. Let R be a commutative ring with 1 and let A,